

**$B \rightarrow J/\psi K$  Decays in QCD Factorization****Hai-Yang Cheng<sup>1,2</sup> and Kwei-Chou Yang<sup>3</sup>**<sup>1</sup> Institute of Physics, Academia Sinica  
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Chung-Li, Taiwan 320, Republic of China**Abstract**

The hadronic decays  $B \rightarrow J/\psi K(K^*)$  are interesting because experimentally they are the only color-suppressed modes which have been measured, and theoretically they are calculable by QCD factorization even the emitted meson  $J/\psi$  is heavy. We analyze the decay  $B \rightarrow J/\psi K$  within the framework of QCD factorization. We show explicitly the scale and  $\gamma_5$ -scheme independence of decay amplitudes and infrared safety of nonfactorizable corrections at twist-2 order. Leading-twist contributions from the light-cone distribution amplitudes (LCDAs) of the mesons are too small to accommodate the data; the nonfactorizable corrections to naive factorization are small. We study the twist-3 effects due to the kaon and find that the coefficient  $a_2(J/\psi K)$  is largely enhanced by the nonfactorizable spectator interactions arising from the twist-3 kaon LCDA  $\phi_\sigma^K$ , which are formally power-suppressed but chirally, logarithmically and kinematically enhanced. Therefore, factorization breaks down at twist-3 order. Higher-twist effects of  $J/\psi$  are briefly discussed. Our result also resolves the long-standing sign ambiguity of  $a_2(J/\psi K)$ , which turns out to be positive for its real part.

## I. INTRODUCTION

There are several reasons why the decays  $B \rightarrow J/\psi K$  and  $J/\psi K^*$  are of great interest. Experimentally, they are the only color suppressed modes in hadronic  $B$  decays that have been measured so far. These decays receive large nonfactorizable corrections, or equivalently, the conventional parameter  $a_2(J/\psi K)$  is large, of order  $0.20 - 0.30$  [1]. In principle, the magnitude of analogous  $a_2$  also can be extracted directly from the decays  $\bar{B}^0 \rightarrow D^{(*)0} \pi^0(\rho^0)$  and indirectly from the data of  $B^- \rightarrow D^{(*)} \pi(\rho)$  and  $\bar{B}^0 \rightarrow D^{(*)} \pi(\rho)$ . However, the former color-suppressed decay modes of the neutral  $B$  meson are not yet measured. Besides the form factors, the extraction of  $a_2$  from  $B \rightarrow D^{(*)} \pi(\rho)$  depends on the unknown decay constants  $f_D$  and  $f_{D^*}$ . On the contrary, the decay constant  $f_{J/\psi}$  is well determined and the quality of the data for  $B \rightarrow J/\psi K^{(*)}$  has been significantly improved over past years.

From the theoretical point of view, the prominent question is how to calculate the parameter  $a_2(J/\psi K)$ . In the literature, this decay mode has been calculated using QCD sum rules and the hard scattering approach. Khodjamirian and Rückl [2] have applied light-cone sum rules to study the nonfactorizable effects in  $B \rightarrow J/\psi K_S$  and concluded that  $a_2(J/\psi K)$  is negative, whereas Li and Yeh [3] found a positive  $a_2(J/\psi K)$  based on the perturbative QCD hard scattering approach. Therefore, there is a sign ambiguity for  $a_2(J/\psi K)$ . Although it has been argued that a negative  $a_2(J/\psi K)$  is very unlikely for several reasons [1], it is important to have an independent theory calculation to clarify the sign issue. The QCD-improved factorization approach advocated recently by Beneke, Buchalla, Neubert and Sachrajda [4] is suitable for this purpose. In this approach, nonfactorizable effects in  $B \rightarrow M_1 M_2$  with recoiled  $M_1$  and emitted light meson  $M_2$  are calculable since only hard interactions between the  $(BM_1)$  system and  $M_2$  survive in the heavy quark limit.

The aforementioned QCD factorization method is no longer applicable if the emitted meson is heavy. For example, since the  $D^0$  meson produced in  $\bar{B}^0 \rightarrow \pi^0 D^0$  decay is not a compact object with small transverse extension, it will interact with the  $(B\pi)$  system in the presence of soft interactions. In principle, the parameter  $a_2(\pi D)$  cannot be calculated using QCD factorization, though it has been roughly estimated in [4] by treating the charmed meson as a light meson, which is certainly a dubious approximation. Fortunately, the QCD factorization approach can be applied to  $B \rightarrow J/\psi K$  decay since the transverse size of  $J/\psi$  becomes small in the heavy quark limit. However, a recent study by Chay and Kim [5] indicated that while the factorization method is applicable to  $B \rightarrow J/\psi K$ , the leading-twist contributions are too small to explain the data.

In general, power corrections to QCD factorization are suppressed by a factor of  $\Lambda_{\text{QCD}}/m_b$ . However, as shown in [6] and [4], there exist power corrections which are chirally enhanced. Moreover, there are also some power-suppressed terms at twist-3 order which involve the logarithmically infrared divergence, indicating the dominance of soft gluon exchange. That is, there are twist-3 power corrections which are chirally and logarithmically enhanced. In

the present paper, we find that a twist-3 light-cone distribution amplitude of the kaon can lead to a large spectator interaction, which allows to alleviate the discrepancy between theory and experiment for  $B \rightarrow J/\psi K$ .

The purpose of this work on  $B \rightarrow J/\psi K$  within the framework of QCD factorization is twofold. First of all, we perform similar leading-twist analysis as in [5]. Second, we investigate higher-twist effects on  $a_2(J/\psi K)$  and show that factorization breaks down. The rest of this paper is organized as follows. In Sec. II, we review the QCD factorization approach and introduce twist-2 light-cone distribution amplitudes of mesons. Then we proceed to compute vertex and spectator corrections to  $B \rightarrow J/\psi K$ . In Sec. III we discuss the corrections at twist-3 order and give numerical results and discussions in Sec. IV. The conclusion is given in Sec. V.

## II. $B \rightarrow J/\psi K$ AT LEADING-TWIST ORDER

### A. QCD factorization

Consider the hadronic decay  $B \rightarrow M_1 M_2$  with  $M_1$  being recoiled and  $M_2$ , which is a light meson or a quarkonium, being emitted, it has been shown that the transition matrix element of an operator  $O$  in the effective weak Hamiltonian valid up to corrections of order  $\Lambda_{\text{QCD}}/m_b$  is schematically given by [4]

$$\begin{aligned} \langle M_1 M_2 | O_i | B \rangle &= \langle M_1 | j_1 | B \rangle \langle M_2 | j_2 | 0 \rangle \left[ 1 + \sum r_n \alpha_s^n + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right) \right] \\ &= \sum_j F_j^{BM_1}(m_2^2) \int_0^1 du T_{ij}^I(u) \phi_{M_2}(u) \\ &\quad + \int_0^1 d\xi du dv T^{II}(\xi, u, v) \phi_B(\xi) \phi_{M_1}(v) \phi_{M_2}(u), \end{aligned} \quad (2.1)$$

where  $F^{BM_1}$  is a  $B - M_1$  transition form factor,  $\phi_M$  is the light-cone distribution amplitude, and  $T^I$ ,  $T^{II}$  are perturbatively calculable hard scattering kernels. In the naive factorization approach,  $T^I$  is independent of  $u$  as it is nothing but the meson decay constant. However, large momentum transfer to  $M_2$  conveyed by hard gluon exchange implies a nontrivial convolution with the distribution amplitude  $\phi_{M_2}$ . The second hard scattering function  $T^{II}$ , which describes hard spectator interactions, survives in the heavy quark limit when both  $M_1$  and  $M_2$  are light or when  $M_1$  is light and  $M_2$  is a quarkonium [4].

The factorization formula (2.1) implies that naive factorization is recovered in the  $m_b \rightarrow \infty$  limit and in the absence of QCD corrections. The radiative corrections to naive factorization are calculable since soft gluon interactions between the  $(BM_1)$  system and the  $M_2$  meson are power-suppressed in the heavy quark limit. Consequently, the nonfactorizable contributions to naive factorization are actually amenable in the infinite quark mass limit.

The QCD factorization formula is not applicable to the decay, for example,  $\bar{B}^0 \rightarrow \pi^0 D^0$  where the emitted meson  $D^0$  is heavy so that it is neither small (with size of order  $1/\Lambda_{\text{QCD}}$ ) nor fast and cannot be decoupled from the  $(B\pi)$  system. This is also ascribed to the fact the soft interaction between  $(B\pi)$  and the  $c$  quark of the  $D^0$  meson is not compensated by that between  $(B\pi)$  and the light spectator quark of the charmed meson.

In Sec. II.C we show that the QCD factorization formula is still applicable to  $B \rightarrow J/\psi K$  decay though the emitted  $J/\psi$  is heavy. The point is that the transverse size of  $J/\psi$  becomes small [of order  $1/(m_c \alpha_s)$ ] in the heavy quark limit. Technically, infrared divergences arising from the soft interactions between the  $c$  quark of  $J/\psi$  and  $(BK)$  system and between the  $\bar{c}$  quark and  $(BK)$  compensate. Therefore, the nonfactorizable contributions to  $B \rightarrow J/\psi K$  is infrared safe.

In principle, power corrections are of order  $\Lambda_{\text{QCD}}/m_b$  and hence they can be neglected in the heavy quark limit. Nevertheless, there are some power corrections which can be enormously enhanced and hence cannot be neglected. First of all, the contributions to, for example  $B \rightarrow K\pi$ , from the  $(S - P)(S + P)$  penguin operators are enhanced by the factor

$$\frac{2\mu_\chi}{m_b} = \frac{2m_K^2}{(m_s + m_u)m_b} = \frac{-4\langle\bar{q}q\rangle}{m_b f_K^2} \sim 12 \frac{\Lambda_{\text{QCD}}}{m_b}, \quad (2.2)$$

which is proportional to the quark condensate. Second, it is shown in [6] that the hard spectator interaction in  $B \rightarrow K\pi$  to twist-3 order has the form

$$f_{II} = \frac{4\pi^2}{N_c} \frac{f_K f_B}{F_1^{B\pi}(0) m_B^2} \int_0^1 \frac{d\bar{\rho}}{\bar{\rho}} \phi^B(\bar{\rho}) \int_0^1 \frac{d\xi}{\xi} \phi^K(\xi) \int_0^1 \frac{d\bar{\eta}}{\bar{\eta}} \left[ \phi^\pi(\bar{\eta}) + \frac{2\mu_\chi}{m_b} \phi_p^\pi(\bar{\eta}) \right]. \quad (2.3)$$

Since the twist-3 distribution amplitude  $\phi_p^\pi(\bar{\eta}) \approx 1$ , it does not vanish at the endpoints. Consequently, the logarithmic divergence of the  $\bar{\eta}$  integral implies that  $f_{II}$  is dominated by soft gluon exchange between the spectator quark and quarks that form the emitted pion, indicating that factorization breaks down at twist-3 order. Hence, this power correction is chirally and logarithmically enhanced. We shall see in Sec. III that the hard spectator interaction in  $B \rightarrow J/\psi K$  receives the same chirally enhanced infrared logarithms from twist-3 kaon distribution amplitudes.

## B. Light-cone distribution amplitudes of mesons

Consider the matrix element of nonlocal operators sandwiched between the vacuum and the vector meson  $J/\psi$ :

$$\begin{aligned} \langle J/\psi | \bar{c}_\alpha^a(x) c_\beta^b(0) | 0 \rangle &= \frac{\delta^{ab}}{4N_c} \left\{ \langle J/\psi | \bar{c}(x) c(0) | 0 \rangle + \gamma_5 \langle J/\psi | \bar{c}(x) \gamma_5 c(0) | 0 \rangle + \gamma^\mu \langle J/\psi | \bar{c}(x) \gamma_\mu c(0) | 0 \rangle \right. \\ &\quad \left. - \gamma^\mu \gamma_5 \langle J/\psi | \bar{c}(x) \gamma_\mu \gamma_5 c(0) | 0 \rangle + \frac{1}{2} \sigma^{\mu\nu} \langle J/\psi | \bar{c}(x) \sigma_{\mu\nu} c(0) | 0 \rangle \right\}_{\beta\alpha}, \end{aligned} \quad (2.4)$$

where  $a, b$  are color indices,  $\alpha, \beta$  are indices for Dirac matrices. The leading-twist light-cone distribution amplitudes (LCDAs) of  $J/\psi$  are given by [7]

$$\begin{aligned}\langle J/\psi(P, \lambda) | \bar{c}(x) \gamma_\mu c(0) | 0 \rangle &= f_{J/\psi} m_{J/\psi} \frac{\varepsilon^{*(\lambda)} \cdot x}{P \cdot x} P_\mu \int_0^1 d\xi e^{i\xi P \cdot x} \phi_{\parallel}^{J/\psi}(\xi), \\ \langle J/\psi(P, \lambda) | \bar{c}(x) \sigma_{\mu\nu} c(0) | 0 \rangle &= -i f_{J/\psi}^T (\varepsilon_\mu^{*(\lambda)} P_\nu - \varepsilon_\nu^{*(\lambda)} P_\mu) \int_0^1 d\xi e^{i\xi P \cdot x} \phi_{\perp}^{J/\psi}(\xi),\end{aligned}\quad (2.5)$$

where  $\varepsilon^*$  is the polarization vector of  $J/\psi$ ,  $\xi$  is the light-cone momentum fraction of the  $c$  quark in  $J/\psi$ ,  $f_{J/\psi}$  and  $f_{J/\psi}^T$  are vector and tensor decay constants, respectively, but the latter is scale dependent. The normalization conditions of the twist-2 LCDAs are

$$\int_0^1 d\xi \phi_{\parallel}^{J/\psi}(\xi) = \int_0^1 d\xi \phi_{\perp}^{J/\psi}(\xi) = 1. \quad (2.6)$$

Likewise, at the twist-2 accuracy, the only kaon LCDA which contributes to the matrix element (the relation  $\sigma^{\mu\nu} \otimes \sigma_{\mu\nu} = \sigma^{\mu\nu} \gamma_5 \otimes \sigma_{\mu\nu} \gamma_5$  being applied)

$$\begin{aligned}\langle K^- | \bar{s}_\alpha^a(0) u_\beta^b(x) | 0 \rangle &= \frac{\delta^{ab}}{4N_c} \left\{ \langle K^- | \bar{s}(0) u(x) | 0 \rangle + \gamma_5 \langle K^- | \bar{s}(0) \gamma_5 u(x) | 0 \rangle + \gamma^\mu \langle K^- | \bar{s}(0) \gamma_\mu u(x) | 0 \rangle \right. \\ &\quad \left. - \gamma^\mu \gamma_5 \langle K^- | \bar{s}(0) \gamma_\mu \gamma_5 u(x) | 0 \rangle + \frac{1}{2} \sigma^{\mu\nu} \gamma_5 \langle K^- | \bar{s}(0) \sigma_{\mu\nu} \gamma_5 u(x) | 0 \rangle \right\}_{\beta\alpha},\end{aligned}\quad (2.7)$$

is given by [8]

$$\langle K^-(P) | \bar{s}(0) \gamma_\mu \gamma_5 u(x) | 0 \rangle = -i f_K P_\mu \int_0^1 d\bar{\eta} e^{i\bar{\eta} P \cdot x} \phi^K(\bar{\eta}), \quad (2.8)$$

where  $\bar{\eta}$  is the momentum fraction of the antiquark  $\bar{u}$  in  $K^-$  and  $\int_0^1 \phi^K(\bar{\eta}) d\bar{\eta} = 1$ .

As for the  $B$  meson LCDA, we will follow [4] to choose

$$\langle 0 | \bar{q}_\alpha(x) b_\beta(0) | \bar{B}(p) \rangle|_{x_+=x_\perp=0} = -\frac{i f_B}{4} [(\not{p} + m_B) \gamma_5]_{\beta\gamma} \int_0^1 d\bar{\rho} e^{-i\bar{\rho} p \cdot x} [\phi_1^B(\bar{\rho}) + \not{p}_- \phi_2^B(\bar{\rho})]_{\gamma\alpha}, \quad (2.9)$$

based on the observation that the  $B$  meson is described by two scalar wave functions at the leading order in  $1/m_b$ , where  $\bar{\rho}$  is the momentum fraction carried by the spectator quark of the  $B$  meson. In Eq. (2.9),  $n_- = (1, 0, 0, -1)$  and the normalization conditions are

$$\int_0^1 d\bar{\rho} \phi_1^B(\bar{\rho}) = 1, \quad \int_0^1 d\bar{\rho} \phi_2^B(\bar{\rho}) = 0. \quad (2.10)$$

The leading-twist LCDAs of  $J/\psi$  can be expanded as [7],

$$\begin{aligned}\phi_{\parallel}^{J/\psi}(\xi) &= 6\xi(1-\xi) \left( 1 + \frac{3}{2} a_2^{\parallel} [5(2\xi-1)^2 - 1] \right), \\ \phi_{\perp}^{J/\psi}(\xi) &= 6\xi(1-\xi) \left( 1 + \frac{3}{2} a_2^{\perp} [5(2\xi-1)^2 - 1] \right),\end{aligned}\quad (2.11)$$

where the parameters  $a_2^\parallel$  and  $a_2^\perp$  are defined by the matrix element of a twist-2 conformal operator with conformal spin 3 [7],\* while twist-2 DA  $\phi^K$  can be expanded in terms of Gegenbauer polynomials  $C_n^{3/2}$  [8]:

$$\phi^K(\bar{\eta}, \mu^2) = 6\bar{\eta}(1 - \bar{\eta}) \left( 1 + \sum_{n=1}^{\infty} a_{2n}^K(\mu^2) C_{2n}^{3/2}(2\bar{\eta} - 1) \right), \quad (2.12)$$

with the values of the Gegenbauer moments  $a_n^K$  being available from [8]. For the  $B$  meson, we use [9]

$$\phi_1^B(\bar{\rho}) = N_B \bar{\rho}^2 (1 - \bar{\rho})^2 \exp \left[ -\frac{1}{2} \left( \frac{\bar{\rho} m_B}{\omega_B} \right)^2 \right], \quad (2.13)$$

with  $\omega_B = 0.25$  GeV and  $N_B$  being a normalization constant. This  $B$  meson wave function corresponds to  $\lambda_B = 303$  MeV defined by  $\int_0^1 d\bar{\rho} \phi^B(\bar{\rho})/\bar{\rho} \equiv m_B/\lambda_B$ . This can be understood since the  $B$  meson wave function is peaked at small  $\bar{\rho}$ : It is of order  $m_B/\Lambda_{\text{QCD}}$  at  $\bar{\rho} \sim \Lambda_{\text{QCD}}/m_B$ . Hence, the integral over  $\phi_B(\bar{\rho})/\bar{\rho}$  produces an  $m_B/\Lambda_{\text{QCD}}$  term.

In ensuing calculations we find that the contributions from the  $J/\psi$  LCDA  $\phi_\perp^{J/\psi}$  are proportional to the factor  $(f_{J/\psi}^T m_c)/(f_{J/\psi} m_{J/\psi})$  relative to that from  $\phi_\parallel^{J/\psi}$ . Contracting  $\langle J/\psi | \bar{c}(0) \sigma_{\mu\nu} c(0) | 0 \rangle$  in Eq. (2.4) with  $P^\nu$  and applying the equation of motion and the matrix element

$$\langle J/\psi | \bar{c}(0) \gamma_\mu c(0) | 0 \rangle = f_{J/\psi} m_{J/\psi} \varepsilon_\mu^*, \quad (2.14)$$

we are led to [5]

$$f_{J/\psi}^T m_{J/\psi} = 2 f_{J/\psi} m_c. \quad (2.15)$$

Therefore,

$$\frac{f_{J/\psi}^T m_c}{f_{J/\psi} m_{J/\psi}} = 2 \left( \frac{m_c}{m_{J/\psi}} \right)^2 = 2\xi^2, \quad (2.16)$$

where in the last step we have applied the on-shell relation  $\xi^2 p_{J/\psi}^2 = m_c^2$  for the charmed (not anticharmed) quark in the  $J/\psi$ .

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\*Since the parameters  $a_2^\parallel$  and  $a_2^\perp$  are unknown, we will employ the asymptotic DAs  $\phi_\parallel^{J/\psi}(\xi) = \phi_\perp^{J/\psi}(\xi) = 6\xi(1 - \xi)$  in numerical calculations in Sec. IV. For general discussions in Secs. II and III, we still use Eq. (2.11) for the LCDAs of  $J/\psi$ .

### C. $B \rightarrow J/\psi K$ in QCD factorization

The effective Hamiltonian relevant for  $B \rightarrow J/\psi K$  has the form

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cs}^* [c_1(\mu) O_1(\mu) + c_2(\mu) O_2(\mu)] - V_{tb} V_{ts}^* \sum_{i=3}^{10} c_i(\mu) O_i(\mu) \right\} + \text{h.c.}, \quad (2.17)$$

where

$$\begin{aligned} O_1 &= (\bar{c}b)_{V-A} (\bar{s}c)_{V-A}, & O_2 &= (\bar{s}b)_{V-A} (\bar{c}c)_{V-A}, \\ O_{3(5)} &= (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A(V+A)}, & O_{4(6)} &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}, \\ O_{7(9)} &= \frac{3}{2} (\bar{s}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V+A(V-A)}, & O_{8(10)} &= \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A(V-A)}, \end{aligned} \quad (2.18)$$

with  $O_3$ – $O_6$  being the QCD penguin operators,  $O_7$ – $O_{10}$  the electroweak penguin operators, and  $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$ .

Under naive factorization, the decay amplitude of  $B \rightarrow J/\psi K$  reads

$$A(B \rightarrow J/\psi K) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (a_2 + a_3 + a_5 + a_7 + a_9) f_{J/\psi} m_{J/\psi} F_1^{BK}(m_{J/\psi}^2) (2\varepsilon^* \cdot p_B), \quad (2.19)$$

where  $a_{2i} = c_{2i} + (1/N_c) c_{2i-1}$  and  $a_{2i-1} = c_{2i-1} + (1/N_c) c_{2i}$  in naive factorization and the approximation  $V_{tb} V_{ts}^* \approx -V_{cb} V_{cs}^*$  has been made. The form factor  $F_1^{BK}$  is defined by

$$\langle P'(p') | V_\mu | P(p) \rangle = \left( p_\mu + p'_\mu - \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu \right) F_1(q^2) + \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu F_0(q^2). \quad (2.20)$$

There are two serious problems with the naive factorization approximation. First, the Wilson coefficients  $c_i(\mu)$  and hence  $a_i$  are renormalization scale and  $\gamma_5$ -scheme dependent, whereas the decay constants and form factors are not. Hence, the amplitude (2.19) is not physical. Second, nonfactorizable effects, which play an essential role in color-suppressed modes, are not taken into account.

The aforementioned two difficulties for naive factorization are resolved in the QCD factorization approach in which the inclusion of vertex corrections and hard spectator interactions (see Fig. 1) yields<sup>†</sup>

$$a_2 = c_2 + \frac{c_1}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_1 \left[ -\left( \frac{18}{14} \right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} + \frac{F_0^{BK}(m_{J/\psi}^2)}{F_1^{BK}(m_{J/\psi}^2)} g_I \right],$$

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<sup>†</sup>Using the constant matrices  $\hat{r}_{\text{NDR}}$  and  $\hat{r}_{\text{HV}}$  given in Eqs. (2.18) and (2.19) in [10], it is straightforward to obtain the constant terms in Eq. (2.21) in NDR and HV  $\gamma_5$ -schemes.

$$\begin{aligned}
a_3 &= c_3 + \frac{c_4}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_4 \left[ -\left(\frac{18}{14}\right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} + \frac{F_0^{BK}(m_{J/\psi}^2)}{F_1^{BK}(m_{J/\psi}^2)} g_I \right], \\
a_5 &= c_5 + \frac{c_6}{N_c} - \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_6 \left[ -\left(\frac{6}{18}\right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} + \frac{F_0^{BK}(m_{J/\psi}^2)}{F_1^{BK}(m_{J/\psi}^2)} g_I \right], \\
a_7 &= c_7 + \frac{c_8}{N_c} - \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_8 \left[ -\left(\frac{6}{18}\right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} + \frac{F_0^{BK}(m_{J/\psi}^2)}{F_1^{BK}(m_{J/\psi}^2)} g_I \right], \\
a_9 &= c_9 + \frac{c_{10}}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_{10} \left[ -\left(\frac{18}{14}\right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} + \frac{F_0^{BK}(m_{J/\psi}^2)}{F_1^{BK}(m_{J/\psi}^2)} g_I \right],
\end{aligned} \tag{2.21}$$

where the upper entry of the matrix is evaluated in the naive dimension regularization (NDR) scheme and the lower entry in the 't Hooft-Veltman (HV) renormalization scheme,  $C_F = (N_c^2 - 1)/(2N_c)$ , and  $N_c$  is the number of colors. The hard scattering functions  $f_I$  and  $g_I$  arise from the vertex corrections, Figs. 1(a)-1(d), while  $f_{II}$  from the hard spectator interactions Figs. 1(e)-1(f). Formally, the coefficients  $a_i$  are scale and  $\gamma_5$ -scheme independent and we will come to this point again in Sec. IV.

The results for the hard scattering functions  $f_I$  and  $g_I$  are

$$\begin{aligned}
f_I &= \int_0^1 d\xi \phi^{J/\psi}(\xi) \left\{ \frac{2z\xi}{1-z(1-\xi)} + (3-2\xi-8\xi^2) \frac{\ln \xi}{1-\xi} \right. \\
&\quad + \left( -\frac{3}{1-z\xi} + \frac{1+8\xi}{1-z(1-\xi)} - \frac{2z\xi}{[1-z(1-\xi)]^2} \right) z\xi \ln z\xi \\
&\quad \left. + \left( 3(1-z) + 2z\xi - 8z\xi^2 + \frac{2z^2\xi^2}{1-z(1-\xi)} \right) \frac{\ln(1-z) - i\pi}{1-z(1-\xi)} \right\},
\end{aligned} \tag{2.22}$$

and

$$\begin{aligned}
g_I &= \int_0^1 d\xi \phi^{J/\psi}(\xi) \left\{ \frac{4\xi(2\xi-1)}{(1-z)(1-\xi)} \ln \xi + \frac{z\xi}{[1-z(1-\xi)]^2} \ln(1-z) \right. \\
&\quad + \left( \frac{1}{(1-z\xi)^2} - \frac{1}{[1-z(1-\xi)]^2} - \frac{8\xi}{(1-z)(1-z\xi)} \right. \\
&\quad \left. \left. + \frac{2(1+z-2z\xi)}{(1-z)(1-z\xi)^2} \right) z\xi \ln z\xi - i\pi \frac{z\xi}{[1-z(1-\xi)]^2} \right\},
\end{aligned} \tag{2.23}$$

where  $z \equiv m_{J/\psi}^2/m_B^2$  and the assumption  $\phi^{J/\psi}(\xi) = \phi_{\parallel}^{J/\psi}(\xi) = \phi_{\perp}^{J/\psi}(\xi)$  has been made. In deriving Eqs. (2.22) and (2.23) we have applied (2.16) and the relation  $\phi^{J/\psi}(\bar{\xi}) = \phi^{J/\psi}(\xi)$ , where  $\bar{\xi} = 1 - \xi$  is the light-cone momentum fraction of the  $\bar{c}$  quark in  $J/\psi$ .

Several remarks are in order. (i) We have proved explicitly the cancellation of infrared divergences so that the resultant amplitude is infrared finite when summing all the diagrams in Fig. 1, a key element for the applicability of QCD factorization. Moreover, we also show a cancellation of the infrared double poles, i.e.  $1/\epsilon_{\text{IR}}^2$ , as in [11]. As stressed and



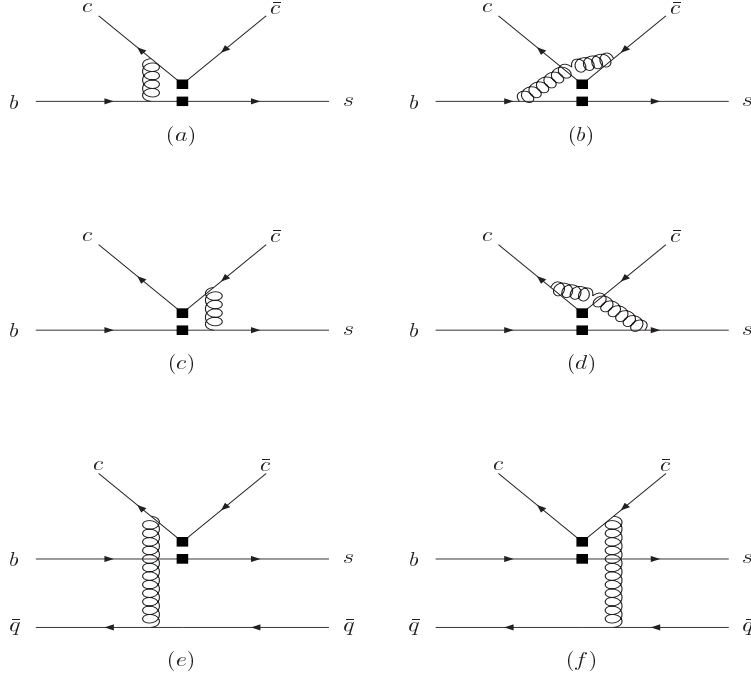


FIG. 1. Vertex and spectator corrections to  $B \rightarrow J/\psi K$ .

elucidated in [5], in the limits  $z \rightarrow 0$  and  $z \neq 0$ , the amplitudes are infrared finite in both cases, but for different reasons. (ii) The strong phases in  $f_I$  and  $g_I$  arise from the diagrams Figs. 1(c)-1(d) where there are hard gluon exchanges between the outgoing  $K$  and  $J/\psi$ . (iii) Only the form factor  $F_1^{BK}$  contributes to the decay amplitude under naive factorization, while to the order  $\alpha_s$  both form factors  $F_0^{BK}$  and  $F_1^{BK}$  contribute. (iv) Our results for  $f_I$  and  $g_I$  agree with that in [5] by noting that the form factors  $F_{0,1}^{BK}$  are related via  $F_0^{BK}(m_{J/\psi}^2)/F_1^{BK}(m_{J/\psi}^2) = (m_B^2 - m_{J/\psi}^2)/m_B^2$  [5]. The only difference is that we treat the ratio  $(f_{J/\psi}^T m_c)/(f_{J/\psi} m_{J/\psi})$  as  $2\xi^2$  [see Eq. (2.16)], while it is considered to be a constant in [5]. (v) It is easily seen that in the zero charmed quark mass limit,

$$f_I + g_I = \int_0^1 d\xi \phi^{J/\psi}(\xi) \left( 3 \frac{1-2\xi}{1-\xi} \ln \xi - 3i\pi \right), \quad (2.24)$$

in agreement with [4] for  $B \rightarrow \pi\pi$ , as it should be.

As for the hard scattering function  $f_{II}$  originating from spectator diagrams, we write

$$f_{II} = f_{II}^2 + f_{II}^3 + \dots, \quad (2.25)$$

where the superscript denotes the twist dimension of LCDA. To the leading-twist order, we obtain

$$f_{II}^2 = \frac{4\pi^2}{N_c} \frac{f_K f_B}{F_1^{BK}(m_{J/\psi}^2 m_B^2)} \int_0^1 d\xi d\bar{\rho} d\bar{\eta} \phi_1^B(\bar{\rho}) \phi^{J/\psi}(\xi) \phi^K(\bar{\eta}) \times \frac{\bar{\rho} - \bar{\eta} + (\bar{\rho} - 2\xi + \bar{\eta})z + 4\xi^2 z}{\bar{\rho}(\bar{\rho} - \bar{\eta} + \bar{\eta}z)[(\bar{\rho} - \xi)(\bar{\rho} - \bar{\eta}) + (\bar{\eta}\bar{\rho} - \bar{\eta}\xi - \bar{\rho}\xi)z]}. \quad (2.26)$$

This can be further simplified by noting that  $\bar{\rho} \sim \mathcal{O}(\Lambda_{\text{QCD}}/m_b) \rightarrow 0$  in the  $m_b \rightarrow \infty$  limit. Hence,

$$f_{II}^2 = \frac{4\pi^2}{N_c} \frac{f_K f_B}{F_1^{BK}(m_{J/\psi}^2 m_B^2)} \frac{1}{1-z} \int_0^1 d\bar{\rho} \frac{\phi_1^B(\bar{\rho})}{\bar{\rho}} \int_0^1 d\xi \frac{\phi^{J/\psi}(\xi)}{\xi} \int_0^1 d\bar{\eta} \frac{\phi^K(\bar{\eta})}{\bar{\eta}}, \quad (2.27)$$

where the  $z$  terms in the numerator cancel after the integration over  $\xi$ .<sup>‡</sup>

Before proceeding we make two remarks: (i) At a first glance, it appears that the spectator contribution is power suppressed by order  $(\Lambda_{\text{QCD}}/m_b)^2$ . This is not the case. First of all, the  $B$  wave function  $\phi_1^B(\bar{\rho})$  is of order  $m_b/\Lambda_{\text{QCD}}$  at  $\bar{\rho} \sim \Lambda_{\text{QCD}}/m_b$  [see also the discussion after Eq. (2.9)]. Second, the form factor  $F_1^{BK}$  scales as  $\sim (\Lambda_{\text{QCD}}/m_b)^{3/2}$ , while decay constants as  $f_B \sim \Lambda_{\text{QCD}}^{3/2}/m_b^{1/2}$  and  $f_\pi \sim \Lambda_{\text{QCD}}$  [4]. It follows that  $f_{II}^2$  is of the same order as  $f_I$  and  $g_I$  and is not power suppressed, contrary to the claim in [5]. Furthermore, the former is numerically more important than the latter, as we shall see in Sec. IV. (ii) By inspection of (2.26), there will be some strong phases coming from hard spectator interactions beyond the heavy quark limit, which are difficult to evaluate numerically, however.

### III. HIGHER-TWIST EFFECTS

In the last section we have computed leading-twist vertex and hard spectator corrections to naive factorization. However, we shall see in Sec. IV that numerically the twist-2 nonfactorizable effects are small; the predicted decay rate of  $B \rightarrow J/\psi K$  is too small by a factor of  $7 \sim 10$ . Therefore, it is inevitable that higher-twist effects which are seemingly power suppressed should play an essential role.

As shown in Sec. II.A, twist-3 power corrections to the spectator interactions could be enormously enhanced by the chirally enhanced infrared logarithms. One reason that the spectator diagrams could receive large power corrections is ascribed to the fact that the hard gluon exchange in the spectator diagram is not hard enough. The virtual gluon's momentum squared is

$$k^2 = (-\bar{\rho}p_B + \bar{\eta}p_K)^2 \approx -\bar{\rho}\bar{\eta}m_B^2 \sim -\Lambda_{\text{QCD}}m_b, \quad (3.1)$$

---

<sup>‡</sup>Although the same expression (2.27) is also given in [5], it is not clear to us how to achieve the cancellation of the  $z$  terms appearing in the numerator of Eq. (2.26) when  $4\xi^2$  is replaced by  $2(f_{J/\psi}^T m_c)/(f_{J/\psi} m_{J/\psi})$  and treated as a constant according to [5].

which is of order 1 GeV due to the smallness of the momentum fraction  $\bar{\rho} \sim \Lambda_{\text{QCD}}/m_b$  carried by the spectator quark in the  $B$  meson. Therefore, we shall study higher-twist power corrections to hard spectator interactions.

### A. Twist-3 LCDAs of the kaon

We first consider the twist-3 DAs  $\phi_p^K$  and  $\phi_\sigma^K$  of the kaon defined in the pseudoscalar and tensor matrix elements [8]:

$$\begin{aligned}\langle K^-(P) | \bar{s}(0) i \gamma_5 u(x) | 0 \rangle &= \frac{f_K m_K^2}{m_s + m_u} \int_0^1 d\bar{\eta} e^{i\bar{\eta} P \cdot x} \phi_p^K(\bar{\eta}), \\ \langle K^-(P) | \bar{s}(0) \sigma_{\mu\nu} \gamma_5 u(x) | 0 \rangle &= -\frac{i}{6} \frac{f_K m_K^2}{m_s + m_u} \left[ 1 - \left( \frac{m_s + m_u}{m_K} \right)^2 \right] \\ &\quad \times (P_\mu x_\nu - P_\nu x_\mu) \int_0^1 d\bar{\eta} e^{i\bar{\eta} P \cdot x} \phi_\sigma^K(\bar{\eta}).\end{aligned}\quad (3.2)$$

They can be expanded in terms of Gegenbauer polynomials [8]:

$$\begin{aligned}\phi_p^K(\bar{\eta}) &= 1 + a C_2^{1/2}(\bar{\eta}) + b C_4^{1/2}(\bar{\eta}) + \dots, \\ \phi_\sigma^K(\bar{\eta}) &= 6\bar{\eta}(1 - \bar{\eta})(1 + d C_2^{3/2}(\bar{\eta}) + \dots),\end{aligned}\quad (3.3)$$

where the coefficients  $a$ ,  $b$ ,  $d$  can be found in [8]. From Eq. (3.2) it is clear that twist-3 DAs of pseudoscalar mesons are associated with a chiral enhancement factor  $\mu_\chi$ . As before,  $\bar{\eta}$  is the light-cone momentum fraction of the  $\bar{u}$  quark in  $K^-$ . Just as the hard spectator interactions in  $B \rightarrow K\pi$  decay which receive large power-suppressed corrections from twist-3 LCDAs of the pion (see Sec. II.A), we find that the twist-3 kaon LCDA  $\phi_\sigma^K$  contributes to spectator diagrams in  $B \rightarrow J/\psi K$  decay and yields

$$\begin{aligned}f_{II}^3 &= \frac{4\pi^2}{N_c} \frac{f_K f_B}{F_1^{BK}(m_{J/\psi}^2 m_B^2)} \frac{2m_K^2}{(m_s + m_u)m_B^2} \int_0^1 d\xi d\bar{\rho} d\bar{\eta} \phi_1^B(\bar{\rho}) \phi^{J/\psi}(\xi) \phi_\sigma^K(\bar{\eta}) \\ &\quad \times \frac{1}{6} \left( \frac{1}{\xi \bar{\rho} \bar{\eta}^2 (1 - z)^3} - \frac{4z}{\bar{\rho} \bar{\eta}^3 (1 - z)^3} + \frac{8\xi z}{\bar{\rho} \bar{\eta}^3 (1 - z)^3} \right).\end{aligned}\quad (3.4)$$

Applying the twist-3 DA  $\phi_\sigma^K(\bar{\eta}) = 6\bar{\eta}(1 - \bar{\eta})$ , we see that the linear divergent terms proportional to  $\int_0^1 d\bar{\eta}/\bar{\eta}^2$  cancel after the integration over  $\xi$ ; this cancellation happens because the  $J/\psi$  distribution amplitude is even in  $(2\xi - 1)$  [see Eq. (2.11)]. Consequently,

$$f_{II}^3 = \left( \frac{2\mu_\chi}{m_B} \right) \frac{4\pi^2}{N_c} \frac{f_K f_B}{F_1^{BK}(m_{J/\psi}^2 m_B^2)} \int_0^1 \frac{d\bar{\rho}}{\bar{\rho}} \phi_1^B(\bar{\rho}) \int_0^1 \frac{d\xi}{\xi} \phi^{J/\psi}(\xi) \int_0^1 \frac{d\bar{\eta}}{\bar{\eta}^2} \frac{\phi_\sigma^K(\bar{\eta})}{6(1 - z)^3}, \quad (3.5)$$

with  $\mu_\chi$  being defined in Eq. (2.2). Therefore, to the twist-3 order of kaon DA,

$$f_{II} = f_{II}^2 + f_{II}^3 = \frac{4\pi^2}{N_c} \frac{f_K f_B}{F_1^{BK}(m_{J/\psi}^2 m_B^2)} \frac{1}{1-z} \int_0^1 \frac{d\bar{\rho}}{\bar{\rho}} \phi_1^B(\bar{\rho}) \int_0^1 \frac{d\xi}{\xi} \phi^{J/\psi}(\xi) \\ \times \int_0^1 \frac{d\bar{\eta}}{\bar{\eta}} \left( \phi^K(\bar{\eta}) + \frac{2\mu_\chi}{m_B} \frac{1}{(1-z)^2} \frac{\phi_\sigma^K(\bar{\eta})}{6\bar{\eta}} \right), \quad (3.6)$$

which, apart from the constant term, agrees with Eq. (2.3) in the  $z \rightarrow 0$  limit.<sup>§</sup> The logarithmic divergence of the  $\bar{\eta}$  integral implies that the spectator interaction is dominated by soft gluon exchanges between the spectator quark and the charmed or anti-charmed quark of  $J/\psi$ . Hence, factorization breaks down to twist-3 order. Following [6], we treat the divergent integral as an unknown parameter and write

$$X \equiv \int_0^1 \frac{d\bar{\eta}}{\bar{\eta}} = \ln \left( \frac{m_B}{\Lambda_{\text{QCD}}} \right) + r, \quad (3.7)$$

with  $r$  being a complex random number. Therefore, although  $f_{II}^3$  is formally power suppressed in the heavy quark limit, it is chirally enhanced by a factor of  $(2\mu_\chi/\Lambda_{\text{QCD}}) \sim 12$ , logarithmically enhanced by the infrared logarithms and kinematically enhanced by a factor of  $1/(1-z)^2$ . Numerically, nonfactorizable corrections to naive factorization are dominated by  $f_{II}^3$  (see Sec. IV).

## B. Higher-twist LCDAs of $J/\psi$

Intuitively it is expected that light mesons produced in hadronic  $B$  decays are appropriately described by LCDAs. Since  $J/\psi$  is not light enough, it is nature to conjecture the importance of higher-twist effects of  $J/\psi$ . The twist-3 LCDAs of  $J/\psi$  are given by [7]

$$\langle J/\psi(P, \lambda) | \bar{c}(x) \gamma_\mu c(0) | 0 \rangle = \cdots + f_{J/\psi} m_{J/\psi} \left( \varepsilon_\mu^{*(\lambda)} - \frac{\varepsilon^{*(\lambda)} \cdot x}{P \cdot x} P_\mu \right) \int_0^1 d\xi e^{i\xi P \cdot x} g_\perp^{(v)}(\xi), \\ \langle J/\psi(P, \lambda) | \bar{c}(x) \gamma_\mu \gamma_5 c(0) | 0 \rangle = -\frac{1}{4} m_{J/\psi} \left( f_{J/\psi} - f_{J/\psi}^T \frac{2m_c}{m_{J/\psi}} \right)$$

---

<sup>§</sup>Just as  $B \rightarrow J/\psi K$  decay, we find that the twist-3 pion distribution amplitude that contributes directly to the spectator interactions of  $B \rightarrow K\pi$  is  $\phi_\sigma^\pi$  rather than  $\phi_p^\pi$ . More specifically, we have a contribution like  $X = \int_0^1 (d\bar{\eta}/\bar{\eta}^2) \phi_\sigma^\pi/6$ . At a first glance, it seems to be different than the last term appearing in Eq. (2.3). However, since the LCDAs  $\phi_p$  and  $\phi_\sigma$  are related by equations of motion [12],  $\phi_\sigma(\bar{\eta})/6 = \bar{\eta}(\phi_p(\bar{\eta})/2 + \phi'_\sigma(\bar{\eta})/12)$ , it follows that  $X = \int_0^1 (d\bar{\eta}/\bar{\eta})(\phi_p(\bar{\eta})/2 + \phi'_\sigma(\bar{\eta})/12)$ . This means that  $X$  can be expressed in terms of  $\phi_p$  and  $\phi'_\sigma$  which do not vanish at endpoints. Since  $\phi'_\sigma(\bar{\eta}) = 6(1-2\bar{\eta})$ ,  $X$  is reduced to  $\int_0^1 (d\bar{\eta}/\bar{\eta}) \phi_p(\bar{\eta})$  after the non-logarithmic term is dropped. Nevertheless, we wish to emphasize that the logarithmic divergence arises originally from  $\phi_\sigma$  instead of  $\phi_p$ .

$$\times \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} P^\alpha x^\beta \int_0^1 d\xi e^{i\xi P \cdot x} g_\perp^{(a)}(\xi), \quad (3.8)$$

for chiral-even DAs  $g_\perp^{(v)}$  and  $g_\perp^{(a)}$ ,

$$\begin{aligned} \langle J/\psi(P, \lambda) | \bar{c}(x) \sigma_{\mu\nu} c(0) | 0 \rangle &= \cdots - i f_{J/\psi}^T (P_\mu x_\nu - P_\nu x_\mu) \frac{\varepsilon^{*(\lambda)} \cdot x}{(P \cdot x)^2} m_{J/\psi}^2 \int_0^1 d\xi e^{i\xi P \cdot x} h_\parallel^{(t)}(\xi), \\ \langle J/\psi(P, \lambda) | \bar{c}(x) c(0) | 0 \rangle &= i \left( f_{J/\psi} - f_{J/\psi}^T \frac{2m_c}{m_{J/\psi}} \right) (\varepsilon^* \cdot x) m_{J/\psi}^2 \int_0^1 d\xi e^{i\xi P \cdot x} h_\parallel^{(s)}(\xi), \end{aligned} \quad (3.9)$$

for chiral-odd DAs  $h_\parallel^{(t)}$  and  $h_\parallel^{(s)}$ , where ellipses denote twist-2 LCDAs given by (2.5).

The calculation of twist-3 effects for  $J/\psi$  is more complicated since it involves four more LCDAs and some other technical problems. Nevertheless, we can argue that the DAs  $g_\perp^{(a)}$  and  $h_\parallel^{(s)}$  are less important. From Eq. (2.16) it follows that the term

$$f_{J/\psi} - \frac{2m_c}{m_{J/\psi}} f_{J/\psi}^T = f_{J/\psi} \left( 1 - \frac{4m_c^2}{m_{J/\psi}^2} \right) \quad (3.10)$$

vanishes in the heavy quark limit. Therefore, it is of order  $\Lambda_{\text{QCD}}/m_b$ . A full study of the twist-3 effects of  $J/\psi$  is in progress. However, just as the  $B \rightarrow K\pi$  decay discussed in Sec. II.A, it is expected that twist-3 corrections which manifest predominately in hard spectator interactions are dominated by the kaon.

#### IV. NUMERICAL RESULTS AND DISCUSSIONS

Before proceeding to the numerical results, we first demonstrate the scale and scheme independence of the coefficients  $a_i$  given in Eq. (2.21). To show the scale independence of  $a_3$ , for example, we note

$$\frac{d c_i(\mu)}{d \ln \mu} = \frac{\alpha_s}{4\pi} \gamma_{ij}^T c_j(\mu), \quad (4.1)$$

where the anomalous dimension matrix  $\gamma$  can be found in the literature (see e.g. [13]). We find

$$\frac{d}{d \ln \mu} (c_3 + \frac{c_4}{3}) = 12 \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_4 \quad (4.2)$$

and hence  $da_3/d \ln \mu = 0$  to  $\mathcal{O}(\alpha_s^2)$ , as it should be.\*\* As for the  $\gamma_5$ -scheme independence of  $a_i$ , we take the NLO Wilson coefficients from [13] (see Table 1). It is ready to check the scheme dependence of  $a_i$  from Eq. (2.21).

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\*\*Empirically one will find that there is a slight scale dependence of  $a_i$  owing to the fact that not all leading logarithmic corrections to  $a_i$  to all orders in  $\alpha_s$  are included in Eq. (2.21).

TABLE I. Lowest-order (LO) and next-to-leading-order Wilson coefficients in NDR and HV  $\gamma_5$ -schemes at  $\mu = \bar{m}_b(m_b) = 4.40$  GeV for  $\Lambda_{\overline{\text{MS}}}^{(5)} = 225$  MeV taken from Table XXII of [13], where  $\alpha$  is the fine-structure constant.

	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7/\alpha$	$c_8/\alpha$	$c_9/\alpha$	$c_{10}/\alpha$
LO	1.144	-0.308	0.014	-0.030	0.009	-0.038	0.045	0.048	-1.280	0.328
NDR	1.082	-0.185	0.014	-0.035	0.009	-0.041	-0.002	0.054	-1.292	0.263
HV	1.105	-0.228	0.013	-0.029	0.009	-0.033	0.005	0.060	-1.283	0.266

From Eq. (2.19) we see that the  $B \rightarrow J/\psi K$  amplitude is governed by the parameter

$$\bar{a}_2 \equiv a_2 + a_3 + a_5 + a_7 + a_9. \quad (4.3)$$

In naive factorization, it is predicted to be  $\bar{a}_2^{\text{LO}}(J/\psi K) = 0.074$  using the leading-order Wilson coefficients at  $\mu = m_b$  given in Table I. Evidently it is too small compared to the experimental value

$$|\bar{a}_2(J/\psi K)|_{\text{expt}} = 0.26 \pm 0.02, \quad (4.4)$$

extracted from the data [14]

$$\mathcal{B}(B^- \rightarrow J/\psi K^-) = (10.0 \pm 1.0) \times 10^{-4}, \quad \mathcal{B}(B^0 \rightarrow J/\psi K^0) = (8.9 \pm 1.2) \times 10^{-4}. \quad (4.5)$$

In ensuing calculations we will use the form factors

$$F_1^{BK}(m_{J/\psi}^2) = 0.70, \quad F_0^{BK}(m_{J/\psi}^2) = 0.50, \quad (4.6)$$

from [1], the decay constants  $f_K = 0.16$  GeV,  $f_B = 0.19$  GeV,  $\Lambda_{\text{QCD}} = 300$  MeV, and the running quark masses [13]:  $m_b = 4.40$  GeV and  $m_c = 1.30$  GeV. For the parameter  $r$  in (3.7), in principle it may be complex due to soft rescattering. In [6],  $r$  is chosen randomly inside a circle in the complex plane of radius 3 (“realistic”) or 6 (“conservative”).

In deriving the hard scattering functions  $f_I$ ,  $g_I$ ,  $f_{II}^2$ ,  $f_{II}^3$ , we have neglected the difference between  $m_b$  and  $m_B$  in the heavy quark limit. However, since  $f_{II}^3$  is sensitive to the value of  $z$ , we will vary  $z$  from one extreme  $z = m_{J/\psi}^2/m_B^2$  to the other extreme  $z = m_{J/\psi}^2/m_b^2$ , as shown in Table II, where we have used  $r = 4.5$  for illustration. The sensitivity of  $f_{II}^3$  on  $z$  is not unexpected because the twist-3 effects on spectator interactions are governed by soft gluon exchange. In Table II we show the numerical values of the parameters  $a_2(J/\psi K)$  and  $\bar{a}_2(J/\psi K)$ , and, for comparison, the twist-2 results of  $\bar{a}_2(J/\psi K)$  denoted by  $\bar{a}_2^{\text{t}2}(J/\psi K)$ , i.e. the predictions without the twist-3 effect  $f_{II}^3$ .

Since *a priori* the shape of the  $J/\psi$  LCDA is unknown, it is worth considering other possibilities besides the asymptotic form which we have used thus far. The delta-function

form  $\phi^{J/\psi}(\xi) = \delta(\xi - \frac{1}{2})$ , as suggested in [5], appeals to the naive expectation of the wave function. The results are shown in the last row in Table II. We see that while  $f_I$  and  $g_I$  are not sensitive to the change of the  $J/\psi$  wave function, the values of  $f_{II}^2$  and  $f_{II}^3$  are reduced by a factor of 2/3 owing to the fact that the integral  $\int_0^1 (d\xi/\xi) \phi^{J/\psi}(\xi)$  [see Eq. (3.6)] is equal to 3 for the asymptotic form and 2 for the delta form of the  $J/\psi$  wave function. As a consequence, the resultant  $|\bar{a}_2(J/\psi K)|$  becomes smaller.

TABLE II. Numerical results of hard scattering functions and the parameters  $\bar{a}_2^{t2}(J/\psi K)$ ,  $a_2(J/\psi K)$ ,  $\bar{a}_2(J/\psi K)$  for two choices of  $z$  and for  $r = 4.5$ . For comparison, the predictions using the delta-function form for the  $J/\psi$  distribution amplitude are shown in the last row.

	$f_I$	$g_I$	$f_{II}^2$	$f_{II}^3$	$\bar{a}_2^{t2}(J/\psi K)$	$a_2(J/\psi K)$	$\bar{a}_2(J/\psi K)$
$z = m_{J/\psi}^2/m_B^2$	-1.17-i6.14	0.54-i0.74	4.89	12.71	0.051-i0.057	0.166-i0.056	0.158-i0.057
$z = m_{J/\psi}^2/m_b^2$	-1.08-i4.60	0.73-i1.27	6.36	21.47	0.065-i0.047	0.254-i0.046	0.247-i0.047
$z = m_{J/\psi}^2/m_B^2$	-1.03-i6.43	0.77-i0.79	3.26	8.47	0.040-i0.059	0.119-i0.059	0.111-i0.059

From Table II we see that at twist-2 order,  $|\bar{a}_2^{t2}(J/\psi K)|$  is of order  $0.07 \sim 0.08$  which is very close to the naive prediction  $a_2^{LO}(J/\psi K) = 0.074$ . This means that nonfactorizable corrections in the heavy quark limit to naive factorization is small. Therefore, the predicted branching ratio of  $B \rightarrow J/\psi K$  to the leading-twist order is too small by a factor of  $7 \sim 10$ . When the unknown parameter  $r = |r| \exp(i\delta)$  is varied from 3 to 6 in the complex plane and  $z$  varies from  $m_{J/\psi}^2/m_B^2$  to  $m_{J/\psi}^2/m_b^2$ , we find that  $|a_2(J/\psi K)|$  and  $|\bar{a}_2(J/\psi K)|$  fall into the range  $0.12 \sim 0.29$  for  $\delta \leq 45^\circ$  (see Fig. 2), which is a reasonable choice since the phase of the spectator diagrams is expected to be small.

Needless to say, the unknown parameter  $X$ , the dependence of  $f_{II}^3$  on  $z$  and the unknown  $J/\psi$  LCDAs are the main sources of theoretical uncertainties. Hence, our result for  $a_2(J/\psi K)$  should be regarded as an order of magnitude estimate. Nevertheless, it is clear that the discrepancy between theory and experiment is greatly improved by the inclusion of kaon twist-3 effects and the sign of  $\text{Re} a_2(J/\psi K)$  is positive. It remains to study the higher-twist effect of  $J/\psi$ .

## V. CONCLUSIONS

We have studied the decay  $B \rightarrow J/\psi K$  within the framework of the QCD factorization approach. Physically, the factorization method is still applicable because the transverse size of  $J/\psi$  becomes small in the heavy quark limit so that its overlap with the  $(BK)$  system is small. Technically, the infrared divergence due to soft gluon exchange between the  $c$  quark and the  $(BK)$  system is canceled by that between the  $\bar{c}$  quark and  $(BK)$  so that

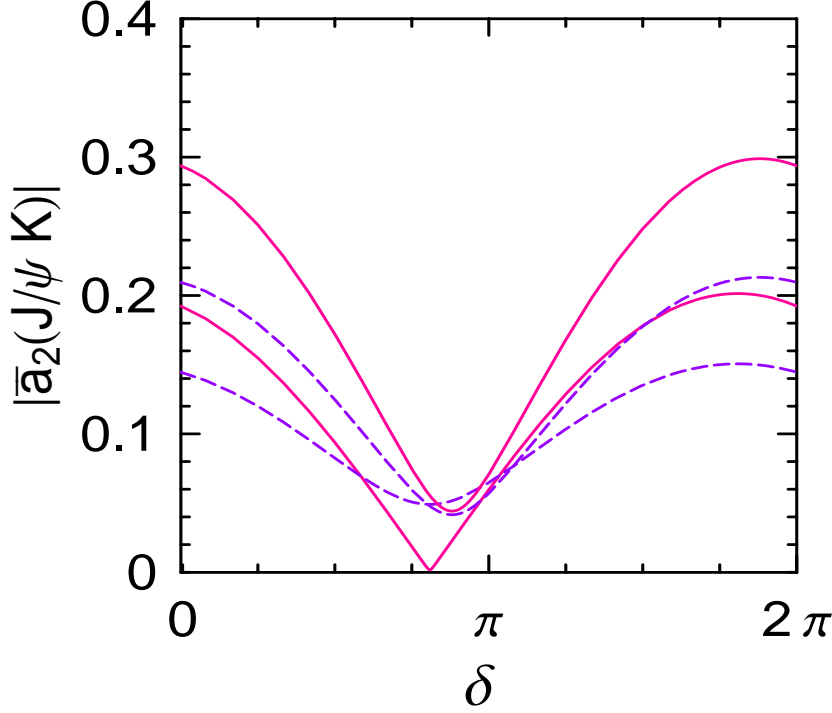


FIG. 2. The coefficient  $|\bar{a}_2(J/\psi K)|$  vs. the phase of the parameter  $r$ . Solid and dashed curves are for  $|r| = 6$  and  $3$ , respectively. The upper and lower solid curves are for  $z = m_{J/\psi}^2/m_b^2$  and  $m_{J/\psi}^2/m_B^2$ , respectively, and likewise for the dashed curves.

nonfactorizable corrections are infrared safe. We have shown explicitly that this is indeed the case to the twist-2 order.

The scale and  $\gamma_5$ -scheme problems with naive factorization are resolved when radiative vertex corrections to the hadronic matrix elements are included, as we have proved explicitly. However, nonfactorizable corrections to naive factorization due to vertex diagrams and hard spectator interactions are small. Therefore, the predicted branching ratio of  $B \rightarrow J/\psi K$  to the leading-twist order is too small by a factor of  $7 \sim 10$  compared to experiment.

Since the virtuality of the hard exchanged gluon in the spectator diagrams is only of order  $m_b \Lambda_{\text{QCD}}$ , it is expected that power-suppressed terms arising from higher-twist wave functions become important. We found that the contribution from the twist-3 kaon light-cone distribution amplitude  $\phi_\sigma^K$  to spectator interactions is not only chirally but also logarithmically and kinematically enhanced, indicating that soft gluon exchange between the spectator quark of the  $B$  meson and the charmed or anticharmed quark of the charmonium dominates. Consequently, factorization breaks down at twist-3 level. In spite of many theoretical uncertainties, the predicted  $|a_2(J/\psi K)|$  can range from 0.12 to 0.29. This also solves the long-standing sign ambiguity for  $\text{Re} a_2(J/\psi K)$ . Though it remains to check the higher-twist effects of the charmonium, it is expected that twist-3 corrections which manifest predominately in hard



spectator interactions are dominated by the kaon.

### **ACKNOWLEDGMENTS**

We are grateful to J. Chay for interesting discussion. One of us (H.Y.C.) wishes to thank the hospitality of the Physics Department, Brookhaven National Laboratory. This work was supported in part by the National Science Council of R.O.C. under Grant Nos. NSC89-2112-M-001-082 and NSC89-2112-M-033-014.

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